THEORETICAL AND EXPERIMENTAL STUDY OF A TURBULENT PLASMA JET

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UDC 539.951.7

A computational technique and the results of an experimental study of a turbulent submerged plasma jet are presented.

A study of the structure of a turbulent jet of low-temperature plasma showed that the total flow field can be divided into three characteristic regions – initial, transition, and main regions [1]. Each of these regions is characterized by its own behavior and it is therefore impossible to obtain a single solution for the entire flow field. Investigation of the initial portion of a plasma jet was the concern of [2-8]. In [2-4], a solution of the problem of the initial portion of a plane-parallel jet was based on the differential equations for a turbulent jet boundary layer. In [7, 8], the same problem for a plane-parallel jet was solved by means of the integral relations for a turbulent jet boundary layer. In [5], an experimental determination was made of the dynamic pressure profiles, temperatures, velocities, and concentrations for a jet of argon plasma emerging into an accompanying flow of cold helium. Both the initial and main portions were included in the experiments. Similar measurements were made [6] for a jet of nitrogen plasma emerging into stationary air.

A Töpler photograph of a jet of water plasma is shown in [4] and a Töpler photograph of a jet of argon plasma in [7]. In both cases, emission was into stationary air with the jet boundary, as determined from Töpler photographs, being rectilinear. Results of studies of the main portion of a plasma jet are given in [2, 5, 6, 9, 10].

Profiles of dynamic pressures, velocities, temperatures, and changes in dynamic pressure and enthalpy along the jet axis are given in [2]. Measurements of temperature and velocity profiles and the axial variation of dynamic pressure and temperature in an air plasma jet are given in [9]. Results of a theoretical analysis of the main portion based on the use of integral relations are also given. Profiles of dynamic pressures in a hydrogen plasma jet are given in [10]. The possibility of using the method for the equivalent problem in the theory of heat conductivity for the analysis of plasma jets is pointed out.

There are practically no papers dealing with the analysis of flow in the transition region of a plasma jet. At the same time, it is impossible to join the solutions for the main portion and the initial portion of a plasma jet correctly without a solution of the problem for the transition region.

In this paper, an attempt is made to solve the problem of the transition region of a plasma jet by a method developed in [1], and based on it, a computational method for a plasma jet is proposed for the entire flow field.

Analysis of Plasma Jet

We consider a subsonic, turbulent, submerged plasma jet flowing out of a circular cylindrical nozzle. A diagram of the propagation of such a jet is shown in Fig. 1. Section 0–0 is the tip of the nozzle. The velocity and enthalpy profiles in this section are assumed to be constant. Section H–H separates the initial portion with a kernel of constant parameters from the transition region. Section P–P separates the transition region from the main region. The jet boundaries in all regions are assumed to be rectilinear.

S. P. Korolev Aviation Institute, Kuibyshev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 17, No. 3, pp. 437-446, September, 1969. Original article submitted October 22, 1968.

Analysis is based on integral relations for a jet boundary layer using Schlichting profiles. Consideration of dissociation and ionization is accomplished by using in place of the equation of state an isobaric relation between density and enthalpy obtained from an analytic approximation to data taken from [1]. This procedure, given in [12], has been used in a number of papers [2-4, 7, 8].

Initial Portion. Under the assumptions made above, the jet can be considered as isobaric. Thus the equation for conservation of momentum written for sections 0–0 and H–H will have the form

$$\pi y_0^2 \rho_0 U_0^2 = 2\pi \int_0^{y_{2H}} \rho U^2 y dy. \tag{1}$$

This equation, the jet equation

$$y_{2H} = c_H k_H x_H, \tag{2}$$

the Schlichting profiles for velocity and enthalpy in the initial portion

$$\frac{U_0 - U}{U_0} = (1 - \eta^{1.5})^2, \tag{3}$$

$$\frac{I_0 - I}{I_0 - I_2} = 1 - \eta \tag{4}$$

and the equation for the isobaric relation between ρ and I

$$\rho = \frac{A}{I^n} \tag{5}$$

make it possible to solve the problem of the geometric structure of the initial region.

According to [1], the coefficient k_H can be determined from

$$k_H = \frac{U_0}{U_{av}}, \tag{6}$$

where the average velocity U_{av} is calculated from:

$$U_{av} = \frac{\int_0^{y_{2H}} \rho U^2 y dy}{\int_0^{y_{2H}} \rho U y dy}. \tag{7}$$

Using the fact that $I_0 \gg I_2$ in all that follows, we find from Eqs. (1)-(7)

$$k_H = \frac{\frac{2}{2.5-n} + \frac{1}{5-n} - \frac{2}{3.5-n} - \frac{1}{4-n}}{1.68\left(\frac{4}{4-n} + \frac{1}{7-n} + \frac{4}{6.5-n} - \frac{4}{5.5-n} - \frac{4}{5-n} - \frac{1}{8-n}\right)}. \tag{8}$$

The factor 1.68 is introduced in the denominator so that the coefficient k_H goes to one for an isothermal jet (n = 0). For the radial coordinate of the external boundary of the jet in the section H–H and length of the initial portion, we obtain:

$$y_{2H} = \frac{y_0}{\sqrt{2(B_2 - B_1)}},$$

$$x_H = \frac{y_0}{c_H k_H \sqrt{2(B_2 - B_1)}}, \tag{10}$$

where

$$B_1 = \frac{4}{5-n} + \frac{1}{8-n} - \frac{4}{6.5-n}, \tag{11}$$

$$B_2 = \frac{4}{4-n} + \frac{1}{7-n} - \frac{4}{5.5-n}. \tag{12}$$

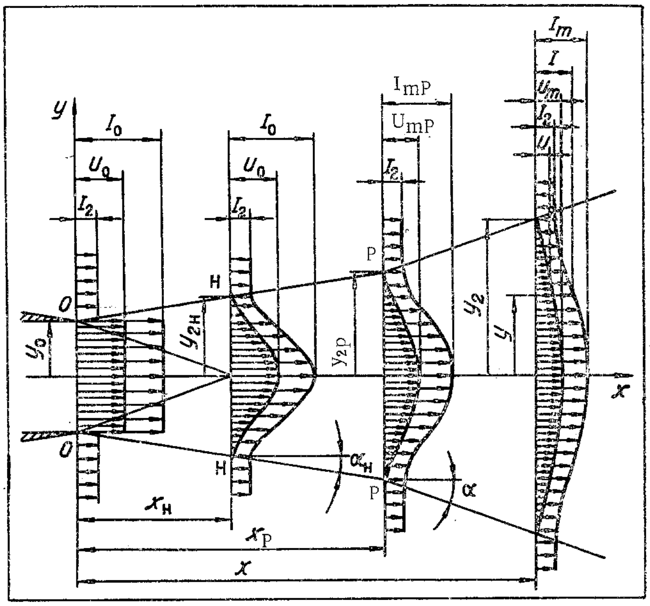


Fig. 1. Diagram of submerged plasma jet propagation.

The angle α_H of the external boundary with respect to the axis of the jet in the initial region is found from the equation

$$\operatorname{tg}\alpha_H = \frac{y_{2H} - y_0}{x_H} = c_H k_H\left[1 - \sqrt{2(B_2 - B_1)}\right]. \tag{13}$$

Equations (3)-(5), (9), and (10) enable one to calculate all geometric and thermodynamic jet parameters in the initial region.

Main Portion. The equations for conservation of momentum and excess enthalpy, written for the section 0–0 and an arbitrary section in the main portion, will have the form:

$$\pi y_0^2 \rho_0 U_0^2 = 2\pi \int_0^{y_2} \rho U^2 y\,dy, \tag{14}$$

$$\pi y_0^2 \rho_0 U_0 (I_0 - I_2) = 2\pi \int_0^{y_2} \rho U (I - I_2)\, y\,dy. \tag{15}$$

Schlichting formulas are used for the velocity and enthalpy profiles in the main portion:

$$\frac{U}{U_m} = (1 - \xi^{1.5})^2, \tag{16}$$

$$\frac{I - I_2}{I_m - I_2} = 1 - \xi^{1.5}. \tag{17}$$

Using Eqs. (5) and (14)-(17) and assuming $I_0 \gg I_2$ and $I_m \gg I_2$, we obtain for the variation of axial parameters along the axis of the jet:

$$\frac{U_m}{U_0} = \frac{B_4}{B_3}\left(\frac{B_3}{2B_4^2}\right)^{\frac{1}{2-n}}\left(\frac{y_0}{y_2}\right)^{\frac{2}{2-n}},$$

$$\frac{I_m}{I_0} = \left(\frac{B_3}{2B_4^2}\right)^{\frac{1}{2-n}}\left(\frac{y_0}{y_2}\right)^{\frac{2}{2-n}}, \tag{19}$$

$$\frac{\rho_m U_m^2}{\rho_0 U_0^2} = \frac{1}{2B_3}\left(\frac{y_0}{y_2}\right)^2, \tag{20}$$

where

$$B_3 = \frac{2(4-n)(3-n)(2-n)\,\Gamma(2-n)\,\Gamma(1.333)}{3(5.333-n)(4.333-n)(3.333-n)(2.333-n)\,\Gamma(2.333-n)}, \tag{21}$$

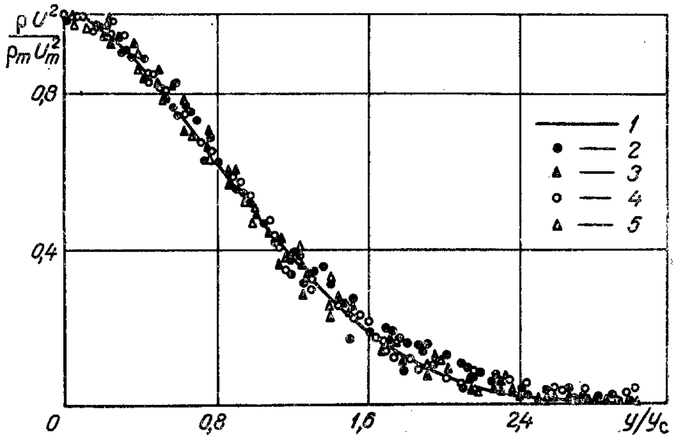


Fig. 2. Dimensionless dynamic pressure profiles in the main portion of turbulent submerged jets: 1) theory; 2) present work, $T_0 = 3900°K$, $x/y_0 = 16$-42; 3) the same, but $T_0 = 4400°K$; 4) from experiments of V. Ya. Bezmenov and V. S. Borisov with air plasma jet [2], $T_0 = 4000°K$, $x/y_0 = 16$-33; 5) from experiments of O'Connor, Comfort, and Cass with nitrogen plasma jet [6], $T_0 = 5800°K$, $x/y_0 = 14$-28.

$$B_4 = \frac{2(3-n)(2-n)\Gamma(2-n)\Gamma(1.333)}{3(4.333-n)(3.333-n)(2.333-n)\Gamma(2.333-n)}. \tag{22}$$

To establish a relation between y_2 and x, we use the jet equation [1]

$$\frac{dy_2}{dx} = c_0 k_0 = \operatorname{tg}\alpha_0. \tag{23}$$

According to [1, 9], the coefficient k_0 can be found from

$$k_0 = \frac{U_m}{U_{av}}, \tag{24}$$

where

$$U_{av} = \int_0^{y_2} \rho U^2 y\,dy \Big/ \int_0^{y_2} \rho U y\,dy. \tag{25}$$

Considering Eqs. (16), (17), (24), and (25), we have

$$k_0 = \frac{(5.333-n)(4.333-n)}{1.93(4-n)(3-n)}. \tag{26}$$

The factor 1.93 in the denominator is introduced for the same reason as the factor 1.68 in the expression for k_H. Since k_0 is independent of x, we obtain after integration of (23)

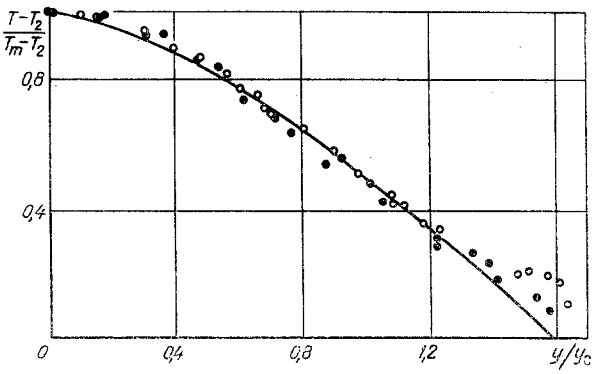
$$y_2 = y_{2P} + c_0 k_0 (x - x_P). \tag{27}$$

This equation is only valid for the main portion. In order to use it, it is necessary to know the radial and longitudinal coordinate of the transition section P–P. This can be accomplished by solving the problem for the transition region.

Transition Portion. The remark has been made [1] that in the case of an incompressible fluid, the external boundary of the transition region is a straight line which is a continuation of the external boundary of the initial portion. In the case of plasma jets, this is confirmed by Töpler photographs [4, 7]. On this basis, the equation

$$y_2 = y_0 + \operatorname{tg}\alpha_H x, \tag{28}$$

which is valid for the initial portion, will also be true for the transition portion.



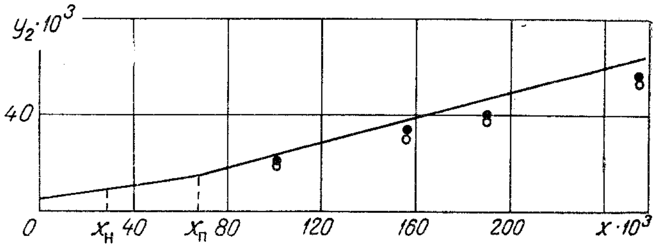


Fig. 3 Fig. 4

Fig. 3. Profiles of dimensionless excess temperatures in the main portion of turbulent sumberged jets: –) theory; ●) present work, T_0 = 3900–1400°K, x/y_0 = 21; ○) experimental results of V. Ya. Bezmenov and V. S. Borisov for an air plasma jet [2], T_0 = 4000°K, x/y_0 = 10–17.

Fig. 4. External jet boundary y_2, m as a function of the longitudinal coordinates x, m: –) theory; ○) present work, T_0 = 3900°K; ●) present work, T_0 = 4400°K.

Within the limits of the transition region, a gradual transition takes place from the behavior in the initial region to the behavior in the main region with the values of the axial parameters of the jet decreasing during the transition from section H–H to section P–P. To establish the nature of this decrease for an incompressible jet, it was recommended [1] that the lines of equal velocity values in the transition portion be considered as a continuation of the corresponding rays in the initial portion. Calculations made on the basis of this proposal for a plasma jet give an increase of dynamic pressure over the length of the transition region, which is in contradiction with the physical picture of the flow. For a plasma jet, it is physically more valid for one to consider that lines of equal dynamic pressure in the transition region are a continuation of the corresponding lines in the initial portion. Using this concept and Eqs. (3)–(5), we obtain for the axial variation of dynamic pressure in the transition region

$$\frac{\rho_m U_m^2}{\rho_0 U_0^2} = \eta_m^{-n}(2\eta_m^{1.5} - \eta_m^3)^2, \tag{29}$$

where $\eta_m = y_2/c_H k_H x$ is a dimensionless axial coordinate in the transition region which is equal to η when $y = 0$; $c_H k_H x$ is that width of boundary layer of the initial portion which it would have propagating to a given cross section in the transition portion. Using Eqs. (28) and (29), we have

$$\frac{\rho_m U_m^2}{\rho_0 U_0^2} = 2\left[\frac{y_2 \operatorname{tg}\alpha_H}{c_H k_H (y_2 - y_0)}\right]^{\frac{3-n}{2}} - \left[\frac{y_2 \operatorname{tg}\alpha_H}{c_H k_H (y_2 - y_0)}\right]^{\frac{6-n}{2}}. \tag{30}$$

If equality of the axial values of dynamic pressure and jet width at section P–P is required as a condition for matching the transition and main regions, we determine $\rho_{mP} U_{mP}^2/\rho_0 U_0^2$ and y_{2P} from a joint solution of Eqs. (20) and (30). This solution is found most simply by a graphical method. The quantities x_P, U_{mP}/U_0, and I_{mP}/I_0 can be found from Eqs. (28), (18), and (19), if the value of y_{2P} is inserted in them in place of y_2. This data is used as input for the calculation of the main portion of the jet.

Note that calculations based on Eq. (29) give values greater than unity for $\rho_m U_m^2/\rho_0 U_0^2$ in the range $0.7 < \eta_m < 1.0$. As shown by an analysis of the data in [1], this is associated with the fact that Eq. (3) gives an overestimate for U/U_0 in comparison for experimental values in the indicated range of variation of η, and Eq. (4) gives an underestimate for I/I_0. However, this situation does not lead to significant error in the determination of jet parameters at the section P–P since the section P–P for plasma jets corresponds to $\eta_m \approx 0.6$, which is a region of satisfactory agreement between Eqs. (3) and (4) and experiment.

Analysis of the equations for conservation of momentum and excess enthalpy leads one to recommend the following approximate expressions for calculating the variation of axial velocity and axial enthalpy over the length of the transition region:

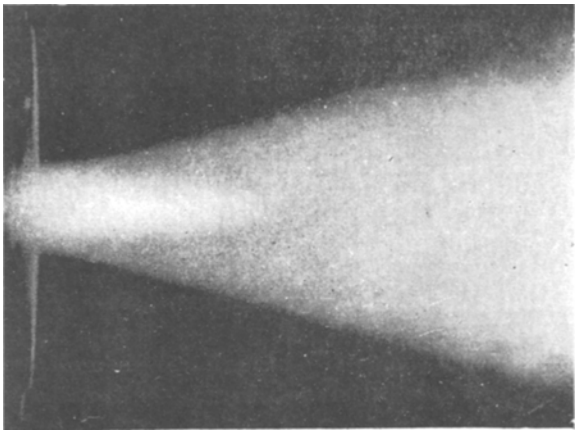


Fig. 5. Topler photograph of argon plasma jet.

$$\frac{U_m}{U_o} \approx \frac{\left(\frac{y_o}{y_2}\right)^{\frac{2}{2-n}}}{\frac{x_P - x}{x_P - x_H}\left(\frac{y_o}{y_{2H}}\right)^{\frac{2}{2-n}} + \frac{x - x_H}{x_P - x_H}\frac{U_o}{U_{mP}}\left(\frac{y_o}{y_{2P}}\right)^{\frac{2}{2-n}}}, \tag{31}$$

$$\frac{I_m}{I_o} \approx \frac{\left(\frac{y_o}{y_2}\right)^{\frac{2}{2-n}}}{\frac{x_P - x}{x_P - x_H}\left(\frac{y_o}{y_{2H}}\right)^{\frac{2}{2-n}} + \frac{x - x_H}{x_P - x_H}\frac{I_o}{I_{mP}}\left(\frac{y_o}{y_{2P}}\right)^{\frac{2}{2-n}}}. \tag{32}$$

The velocity and enthalpy profiles in an arbitrary section of the transition portion can be calculated from the approximate formulas

$$\frac{U}{U_m} \approx \frac{x_P - x}{x_P - x_H}\left[2(1-\xi)^{1.5} - (1-\xi)^3\right] + \frac{x - x_H}{x_P - x_H}(1-\xi^{1.5})^2, \tag{33}$$

$$\frac{I}{I_m} \approx \frac{x_P - x}{x_P - x_H}(1-\xi) + \frac{x - x_H}{x_P - x_H}(1-\xi^{1.5}), \tag{34}$$

which are obtained from the consideration that the profiles should correspond to behavior in the initial portion for $x = x_H$ and to behavior in the main portion for $x = x_P$.

EXPERIMENTAL RESULTS

A study was made of a subsonic air plasma jet emerging from the cylindrical nozzle, 12 mm in diameter, of a constant-current plasmotron. In the experiments, dynamic pressure profiles were measured in transverse sections of the jet located at distances x = 101, 156, 190, and 256 mm from the tip of the nozzle. Sampling of the gas in a water piezometer was accomplished by means of a water-cooled copper probe with an external diameter of 4 mm and an internal diameter of 0.9 mm. Radial variation of gas temperature in the jet was determined only in cross sections most distant from the nozzle tip by L-shaped, uncooled tungsten–tungstorhenium and platinum–platinorhodium thermocouples, the junctions of which were placed along the flow. Thermocouples and cooled probe were moved in the vertical and horizontal directions in the jet flow field by a mechanical device. Measurements were made in a horizontal plane which passed through the point of maximum dynamic pressure.

Dimensionless profiles of dynamic pressure and excess temperature are shown in Figs. 2 and 3 which are satisfactorily approximated by theoretical curves. Other experimental data [2, 6] are also shown for comparison. The temperature profiles shown are equivalent to enthalpy profiles because the temperatures are low in the specified jet cross sections and specific heat can be considered to be constant.

The solid line in Fig. 4 indicates jet boundaries calculated on the assumption of infinitely large initial heating. Also plotted in the same figure are experimental points which show the position of the external boundary of the main portion as determined from our measurements of the dynamic pressure profile. It is clear that the width of the jet increases as the initial heating rises. This conclusion is in agreement with the results of a theoretical calculation for the main region. The experimental points also indicate the rectilinearity of the external jet boundary in the main portion. The existing discrepancy between theoretical and experimental jet boundaries in the main portion is explained by the circumstance that the assumption of an infinitely large initial heating is too crude for the initial temperatures actually occurring in this work.

An evaluation of α_0 based on experimental data of Zabudkina [10], and assuming the pole of the jet is located in the section 0–0, gives good agreement with the theoretical values for α_0. This enables one to consider the theory presented above as valid for jets with an initial temperature above 5000°K.

As confirmation of the assumption in the theory of the reactilinearity of the external boundary of the initial and transition portions of the jet, a Töpler photograph of an argon plasma jet taken from [7] is shown in Fig. 5. The photograph was made with an IAB-451 Töpler tube.

NOTATION

x, y	are the axial and radial coordinates;
U	is the velocity;
T	is the temperature;
I	is the enthalpy;
ρ	is the density;
A, n	are constant depending on the kind of gas and the temperature range;
c_H, c_0	are the experimental constants of initial and main sections, respectively;
α_H, α_0	are the angle of inclination of external boundary to jet axis in initial and main sections, respectively;
k_H, k_0	are the expansion coefficients of jet in initial and main sections, respectively;
Γ	is the gamma-function;
$\eta = (y_2 - y)/(y_2 - y_1)$	is the dimensionless ordinate of initial section;
$\xi = y/y_2$	is the dimensionless ordinate of main and transition sections.

Subscripts

0	refers to conditions at nozzle tip;
H	refers to section H–H;
P	refers to section P–P;
m	refers to jet axis;
1	refers to internal boundary;
2	refers to external boundary;
c	denotes position in jet when properties are half their values on jet axis.

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